Data Mining

Ranking Models

Basics

by

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Information Retrieval

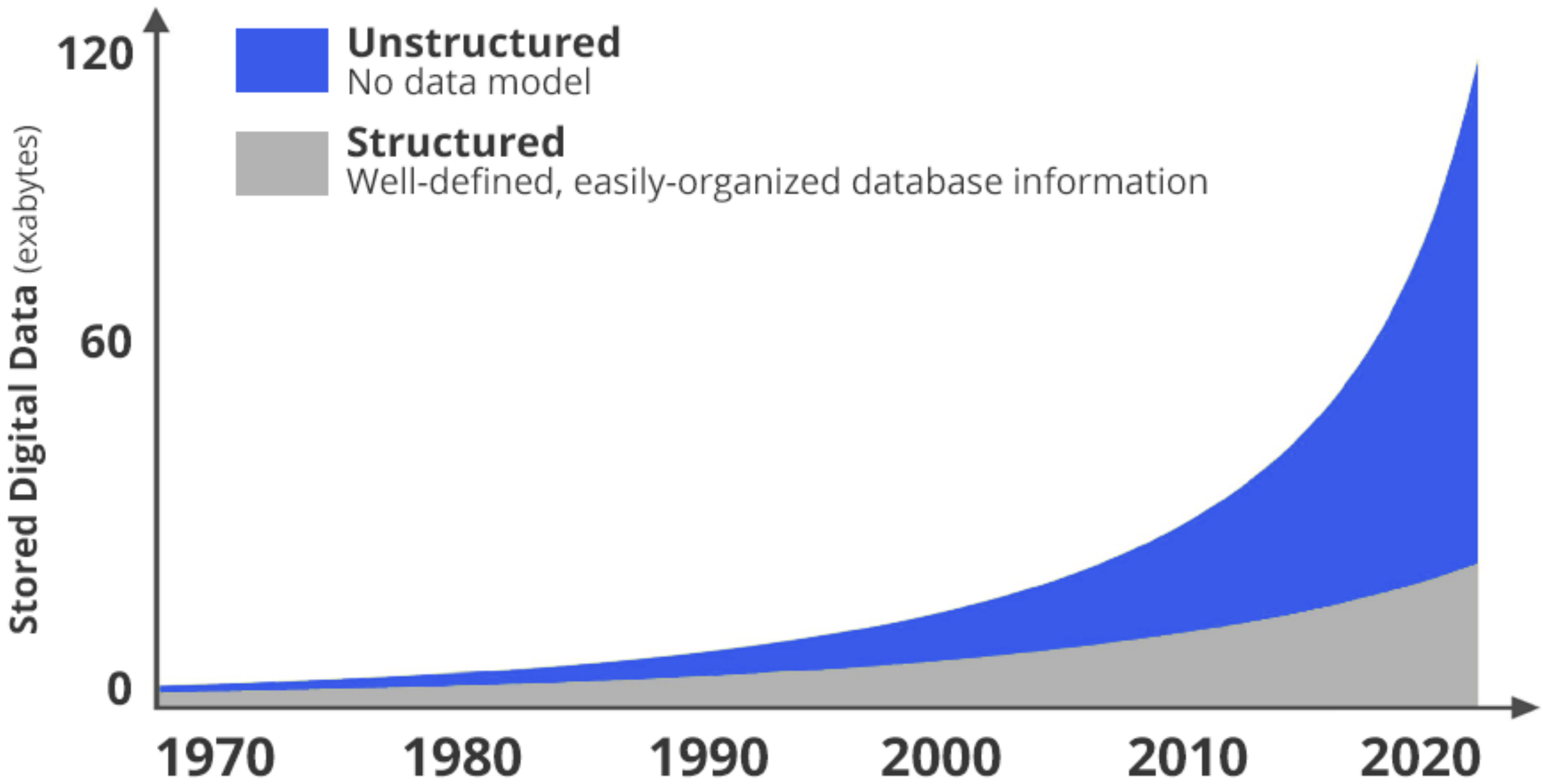
- Information Retrieval (IR) is finding material (usually documents) of an unstructured nature (usually text) that satisfies an information need from large collections (usually stored on computers).

- "*Usually*" text, but can be more: images, videos, data, services, audio..

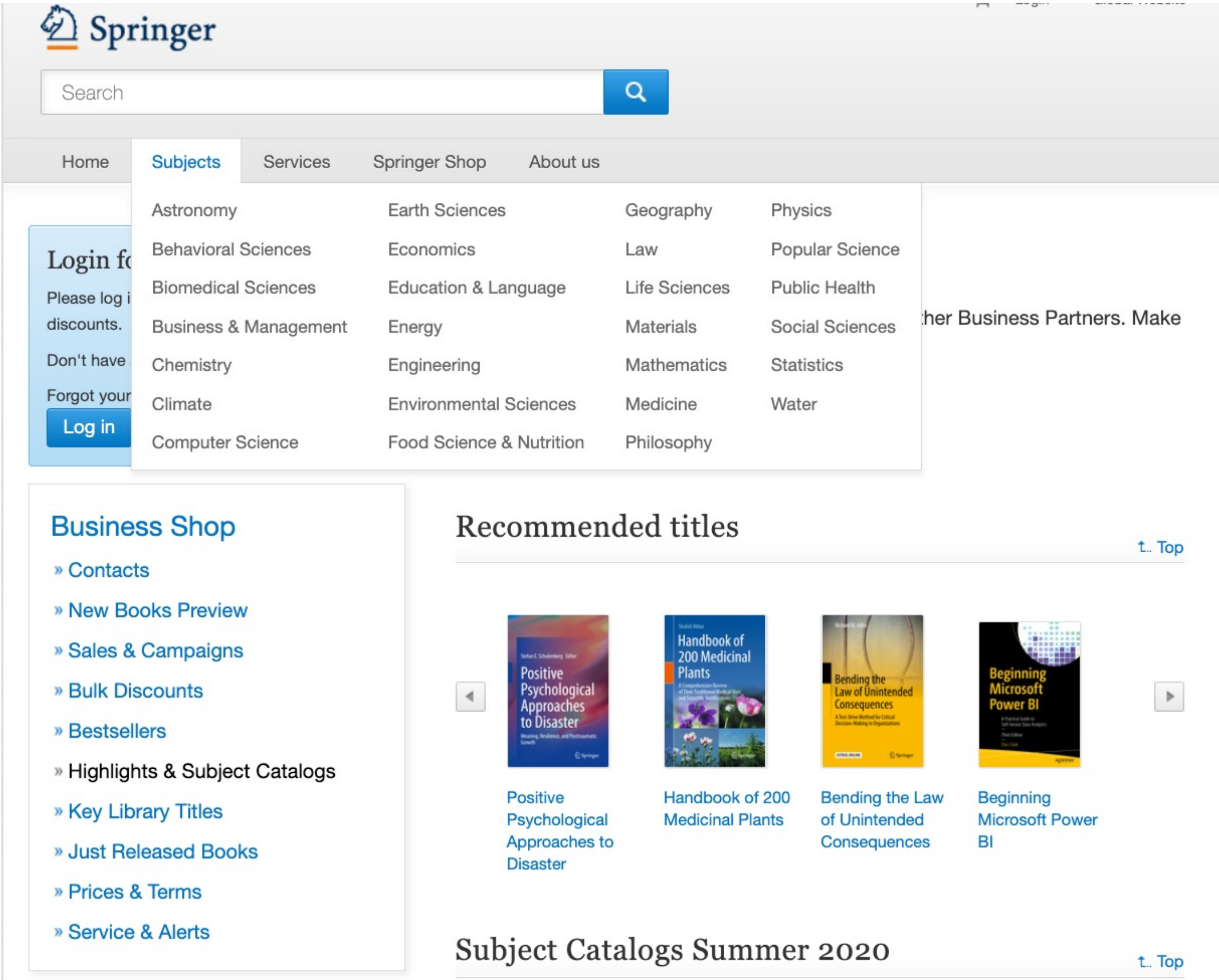
- "*Usually*" unstructured (= no pre-defined model)
  but: XML, RDF, html are "more structured" than txt or pdf

- "*Large*" collections: how large?? The Web! (The Indexed Web contains **at least 50 billion pages** .)

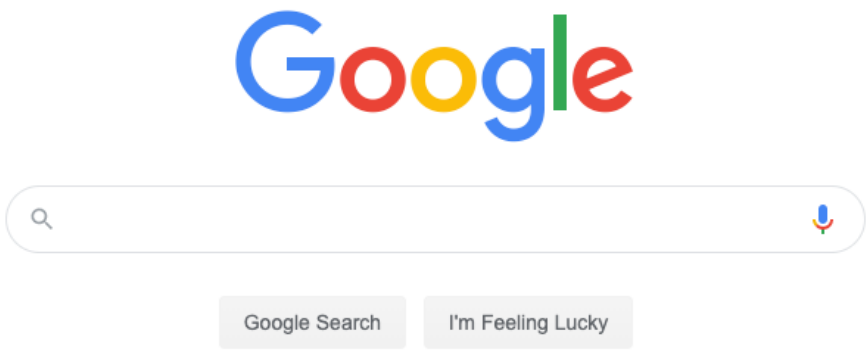
Unstructured vs. structured



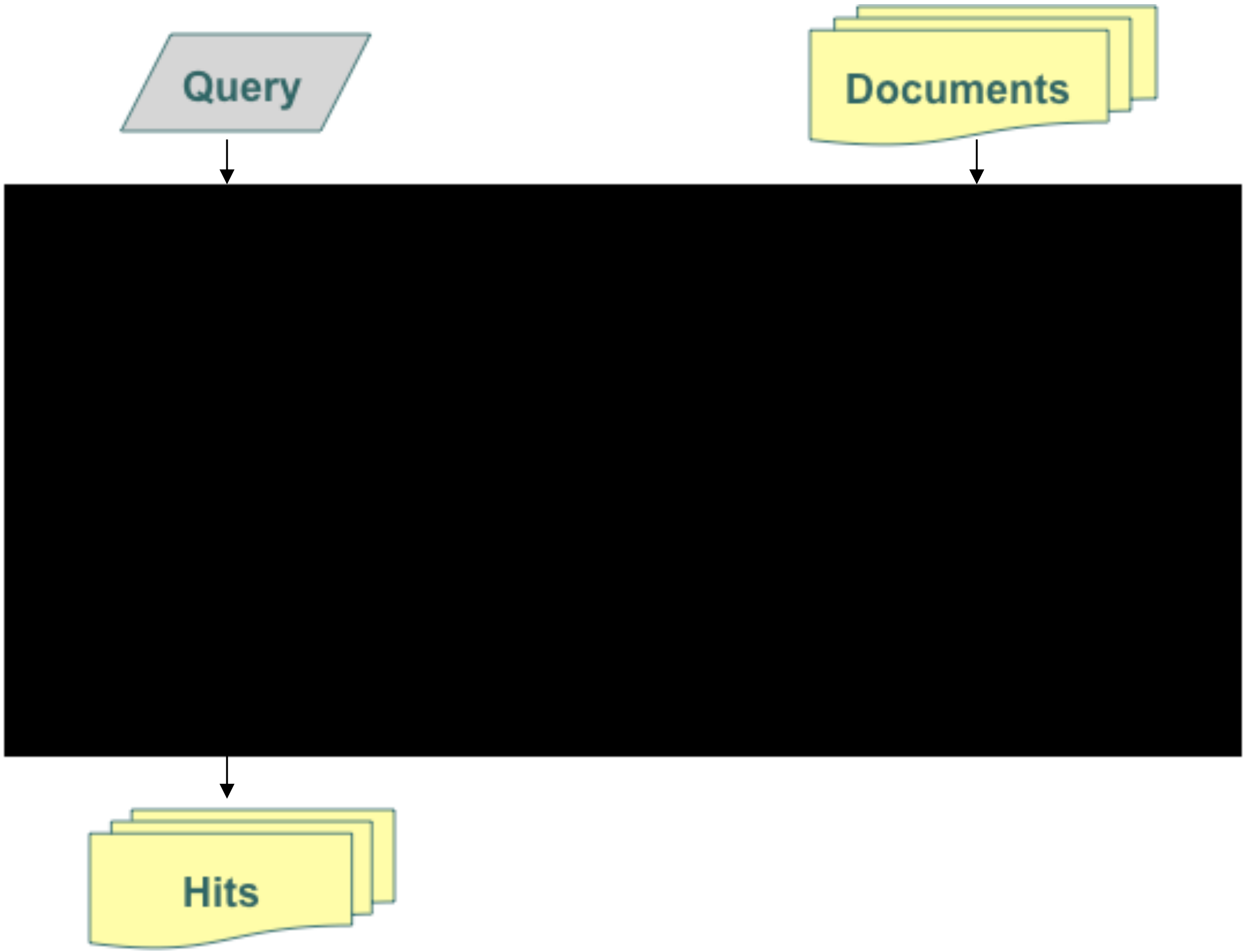
Library/Book Catalogue



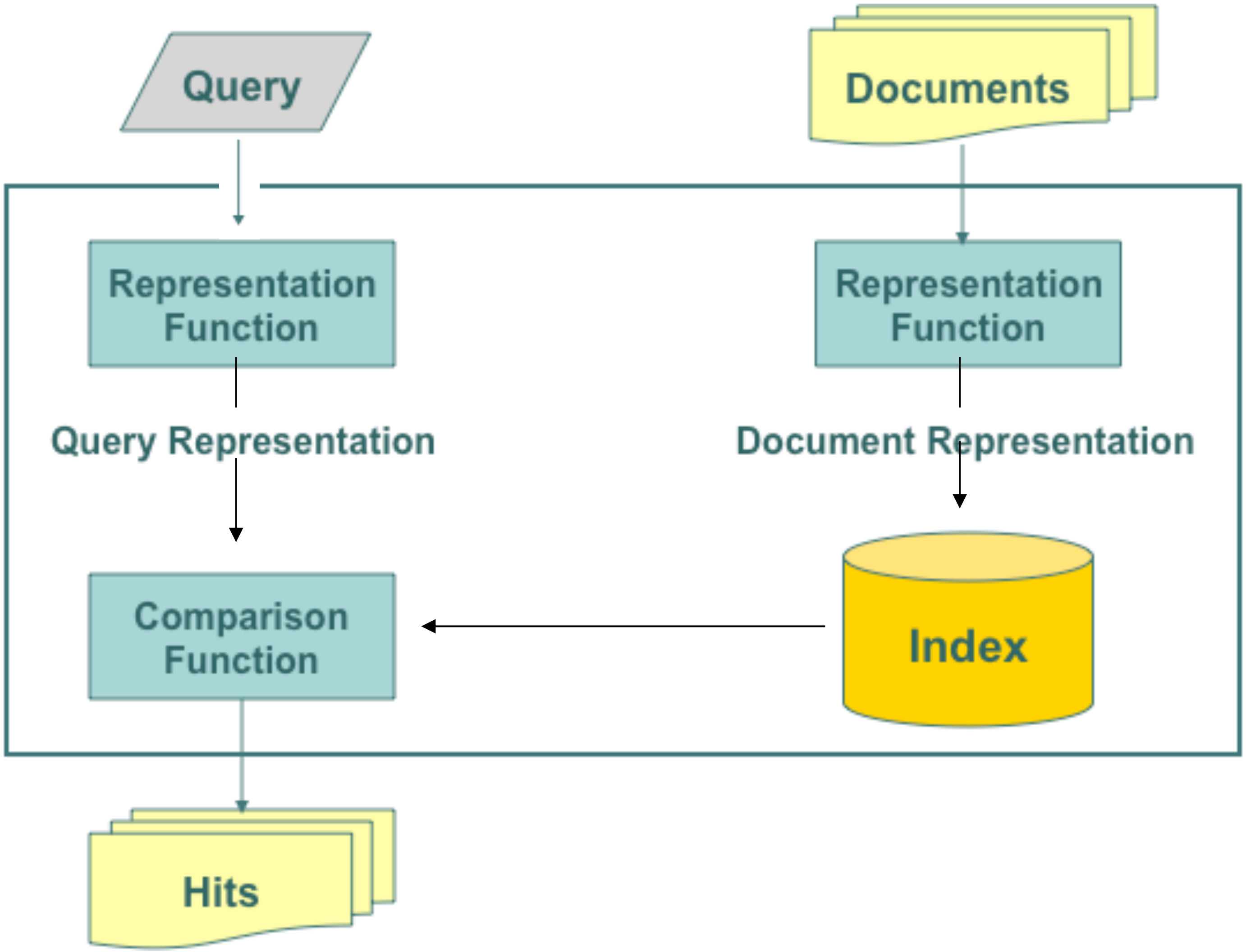
Web Search



IR Black Box



Inside The IR Black Box



Boolean Model

Boolean Model

- Simple model based on set theory
- First model used in "classic" IR systems
- Queries and documents specified as boolean expressions :
  - ❑precise semantics
  - ❑*E.g.,* $q = ka \wedge (kb \vee \neg kc)$
  - ❑*(apple* $\wedge$ *(computer* $\vee$ $\neg$red)
- Terms can be present or absent. Thus, $w_{ij} \in \{0,1\}$

Example

- $q = a \wedge (b \vee (\neg c)) =$

$(a \wedge b \wedge c) \vee (a \wedge b \wedge (\neg c)) \vee (a \wedge (\neg b) \wedge (\neg c))$ *(DNF form)*

- $v(q_{dnf}) = (1,1,1)\ (1,1,0)\ (1,0,0)$
  - » ***Disjunctive Normal Form***
  - » ***Ex: (apple,computer,red) ∨ (apple, computer) ∨ (apple)***
- $v(q_{cc}) = (1,1,0)$
  - » **Conjunctive Component**

Example

- ❑ $q = a \wedge (b \vee (\neg c)) =$

  $(a \wedge b \wedge c) \vee (a \wedge b \wedge (\neg c)) \vee (a \wedge (\neg b) \wedge (\neg c))$ *(DNF form)*

- ❑ $v(q_{dnf}) = (1,1,1)\ (1,1,0)\ (1,0,0)$
  - » ***Disjunctive Normal Form***
  - » ***Ex: (apple,computer,red) ∨ (apple, computer) ∨ (apple)***
- ❑ $v(q_{cc}) = (1,1,0)$
  - » **Conjunctive Component**

- Similar/Matching documents
  - *md1 = [apple apple blue day] => (1,0,0)*
  - *md2 = [apple computer red] => (1,1,1)*
- Unmatched documents
  - *ud1 = [apple red] => (1,0,1)*
  - *ud2 = [day] => (0,0,0)*

Drawbacks of the Boolean Model

- ❑ Expressive power of boolean expressions to capture information needs and document semantics is *inadequate*
- ❑ Retrieval based on binary decision criteria (with no partial match) does not reflect our intuitions behind relevance adequately

- As a result
  - ❑ Answer set (results) contains either too few or too many documents in response to a user query
  - ❑ No ranking of documents

Problem with Boolean search

- Boolean queries often result in either too few (=0) or too many (1000s) results.
  - Query 1: "*standard user dlink 650*" → 200,000 hits
  - Query 2: "*standard user dlink 650 no card found*": 0 hits
- It takes skill to come up with a query that produces a manageable number of hits.

- With **a ranked list of documents**, it does not matter how large the retrieved set is. User will looks at first results.

Vector Model

Scoring as the basis of ranked retrieval

- We wish to return *in order of relevance* the documents most likely to be useful to the searcher
- How can we rank-order the documents in the collection with respect to a query?
- Assign a score – say in [0, 1] – to each document
- This score measures how well document and query "match".

Query-document matching scores

- We need a way of assigning a score to a query/document pair
- Let's start with a one-term query
- If the query term does not occur in the document: score should be 0
- The more frequent the query term in the document, the higher the score (should be)
- We will look at a number of alternatives for this.

Vector Weighting Model

- **Model**: each document is a bag-of-words
- **Representation**:
  a **N-dimensional vector** (N=|V|, the dimension of the vocabulary)
- **Weighting schema**: coordinate w_{ij} of vector d_j associated to document d_j is the RELEVANCE of word i in document j
- How do we measure w_{ij} ?

Bag of words vector

- Vector representation doesn't consider the ordering of words in a document
  - *d1: John is quicker than Mary* and
    d2: Mary is quicker than John
    have the same vectors, since we have a coordinate (or coefficient, or weight) w_i for every word *i* of the vocabulary, and coordinates are ordered alphabetically
  - $d1=d2=(w_{John}, w_{is}, w_{Mary}, w_{quicker}, w_{than})$
- This is called (as mentioned in previous lectures) the bag of words model.
  - In a sense, this is a step back: the **positional index** (see lectures on indexing) was able to distinguish these two documents.

Binary term-document matrix

words	Antony and Cleopatra	Julius Caesar	The Tempest	Hamlet	Othello	Macbeth
			documents			
Antony	1	1	0	0	0	1
Brutus	1	1	0	1	0	0
Caesar	1	1	0	1	1	1
Calpurnia	0	1	0	0	0	0
Cleopatra	1	0	0	0	0	0
mercy	1	0	1	1	1	1
worser	1	0	1	1	1	0

Any column j is a document vector d_j.
Each document is represented by a binary vector $\in \{0,1\}^{|V|}$, w_{ij} is either 0 (word *i* is absent in d_j) or 1 (word *i* appears in d_j)
Number of rows=dimension of vocabulary **|V|**
Number of columns= dimension of the document collection **N**

Term-document count matrix

- This scheme considers the number of occurrences of a term in a document:
  - Each document is a count vector in $\mathbb{N}^{v}$

	Antony and Cleopatra	Julius Caesar	The Tempest	Hamlet	Othello	Macbeth
Antony	157	73	0	0	0	0
Brutus	4	157	0	1	0	0
Caesar	232	227	0	2	1	1
Calpurnia	0	10	0	0	0	0
Cleopatra	57	0	0	0	0	0
mercy	2	0	3	5	5	1
worser	2	0	1	1	1	0

Term frequency tf

- The *term frequency* $tf_{t,d}$ of term *t* in document *d* is defined as the number of times that *t* occurs in *d*.
- We want to use *tf* when computing query-document match scores. But how?
- *Raw* term frequency is *not* what we want:
    - A document with 10 occurrences of the term may be more relevant than a document with one occurrence of the term.
    - But not 10 times more relevant.
- **Relevance does not increase proportionally with term frequency**.
- One possibility is to normalize: $tf_i^{norm} = tf_i / \max_j (tf_j)$

Log-frequency weighting

- The log frequency weight of term t in d is

$$w_{t,d} = \begin{cases} 1 + \log_{10} \mathrm{tf}_{t,d}, & \text{if } \mathrm{tf}_{t,d} > 0 \\ 0, & \text{otherwise} \end{cases}$$

- 0 → 0, 1 → 1, 2 → 1.3, 10 → 2, 1000 → 4, etc.

Scoring similarity

- Score for a document-query pair: sum over terms *t* in both *q* and *d*:
- Sim(q,d) $= \sum_{t \in q \cap d} (1 + \log \mathrm{tf}_{t,d})$
    - **The score is 0 if none of the query terms is present in the document, and grows when the document includes many of the query terms, with a high frequency**
- However, frequency-based ranking (whether normalized or log) IS NOT FULLY APPROPRIATE
- WHY??

Stop words

- With a stop list, you exclude from the dictionary entirely **the commonest words**. Intuition:
  - They have little semantic content: *the, a, and, to, be*
  - There are a lot of them: ~30% of postings for top 30 words
  - Stop word **elimination used to be standard in older IR systems.**

- But the trend is away from doing this:
  - Good compression techniques means the space for including stopwords in a system is very small
  - Good query optimization techniques mean you **pay little** at query time for including stop words.
  - You need them for:
    - Phrase queries: “King of Denmark”
    - Various song/books titles, etc.: “Let it be”, “To be or not to be”
    - “Relational” queries: “flights to London”vrs “flight from London”

Inverse Document Frequency

- **Rare terms are more informative than frequent terms**
  - Recall stop words!
  - Consider a term in the query that is rare in the collection (e.g., *arachnocentric*)
  - A document containing this term is very likely to be relevant to the query *“study on **arachnocentric** people”*
  - → **We want a higher weight for rare terms like *arachnocentric***

Inverse Document Frequency (1)

- Consider a query term that is frequent in the collection (e.g., *high, increase, line*)
  - A document containing such a term is more likely to be relevant than a document that doesn't, *but it's not a sure indicator of relevance.*
  - → For frequent terms, we want lower weights than for rare terms, since they do not characterize a single document

- We will use document frequency (df) to capture the intuition that terms appearing in many documents of the collection should have a lower weight

- df ($\leq N$) = number of documents that contain the term, N= dimension of the document collection

Inverse Document Frequency (2)

- df_t is the <u>document</u> frequency of t:
  the number of documents in the collection that contain t
  - df is a measure of the informativeness of t

- We define the *idf* (inverse document frequency) of t by:

$$\mathrm{idf}_t = \log_{10} N/\mathrm{df}_t$$

  - We use $\log N/\mathrm{df}_t$ instead of N/df_t to "dampen" the effect of idf.

IDF example (N = 1M)

term	df_t = *# of documents including the term*	idf_t
calpurnia	1	6
animal	100	4
sunday	1,000	3
fly	10,000	2
under	100,000	1
the	1,000,000	0

There is one idf value for each term *t* in a collection.

Scoring Similarity: tf-idf

- The tf-idf weight of a term is the product of its tf weight and its idf weight.

$$w_{t,d} = (1 + \log \mathrm{tf}_{t,d}) \times \log N / \mathrm{df}_t$$

- Best known weighting scheme in information retrieval
    - Note: the "-" in tf-idf is a hyphen, not a minus sign!
    - Alternative names: tf.idf, tf x idf
- **Increases** with the number of **occurrences** within a document
- **Increases** with the **rarity** of the term in the collection

Binary → count → weight matrix

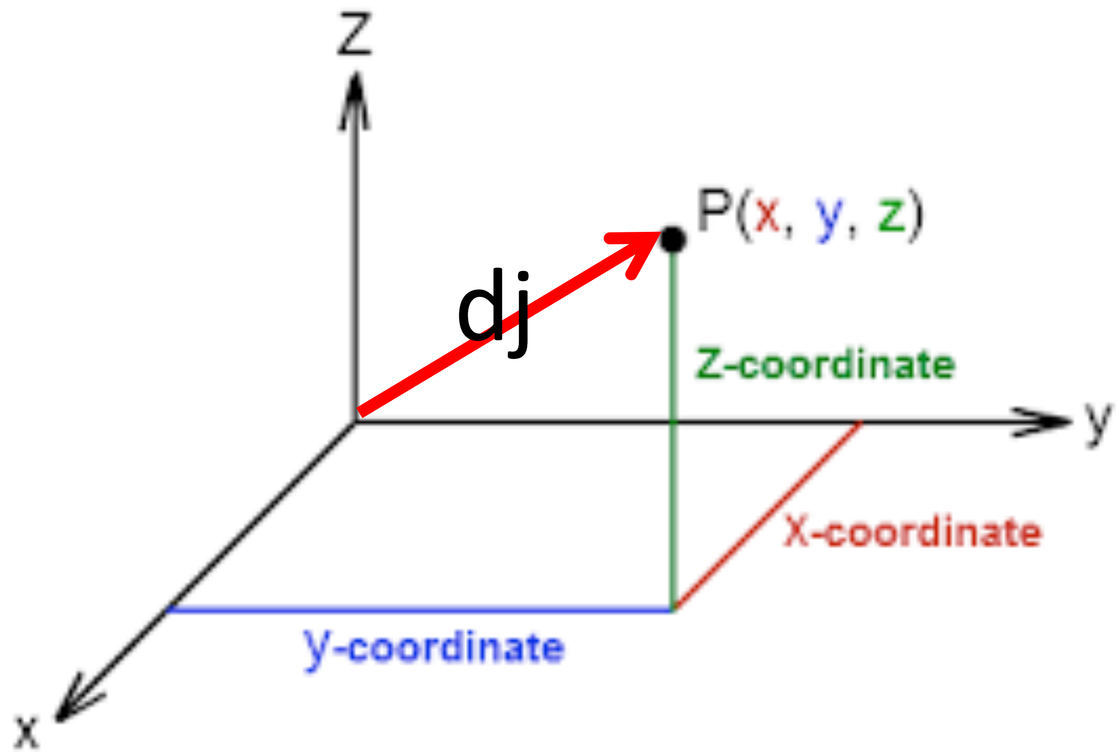
	Antony and Cleopatra	Julius Caesar	The Tempest	Hamlet	Othello	Macbeth
Antony	5.25	3.18	0	0	0	0.35
Brutus	1.21	6.1	0	1	0	0
Caesar	8.59	2.54	0	1.51	0.25	0
Calpurnia	0	1.54	0	0	0	0
Cleopatra	2.85	0	0	0	0	0
mercy	1.51	0	1.9	0.12	5.25	0.88
worser	1.37	0	0.11	4.15	0.25	1.95

Each document is now represented by a real-valued vector of tf-idf weights $\in \mathbb{R}^{|V|}$

Documents as vectors

- So we have a |V|-dimensional vector space, one dimension for each term.
- Terms are axes of the space
- Documents are points or vectors in this space.
- The coordinate of a vector d_j on dimension *i* is the tf-idf weight of word *i* in document *j*.
- **Very high-dimensional**: hundreds of millions of dimensions when you apply this to a web search engine
- It is a very sparse vector - most entries are zero (will see later in this course how to reduce dimensionality).

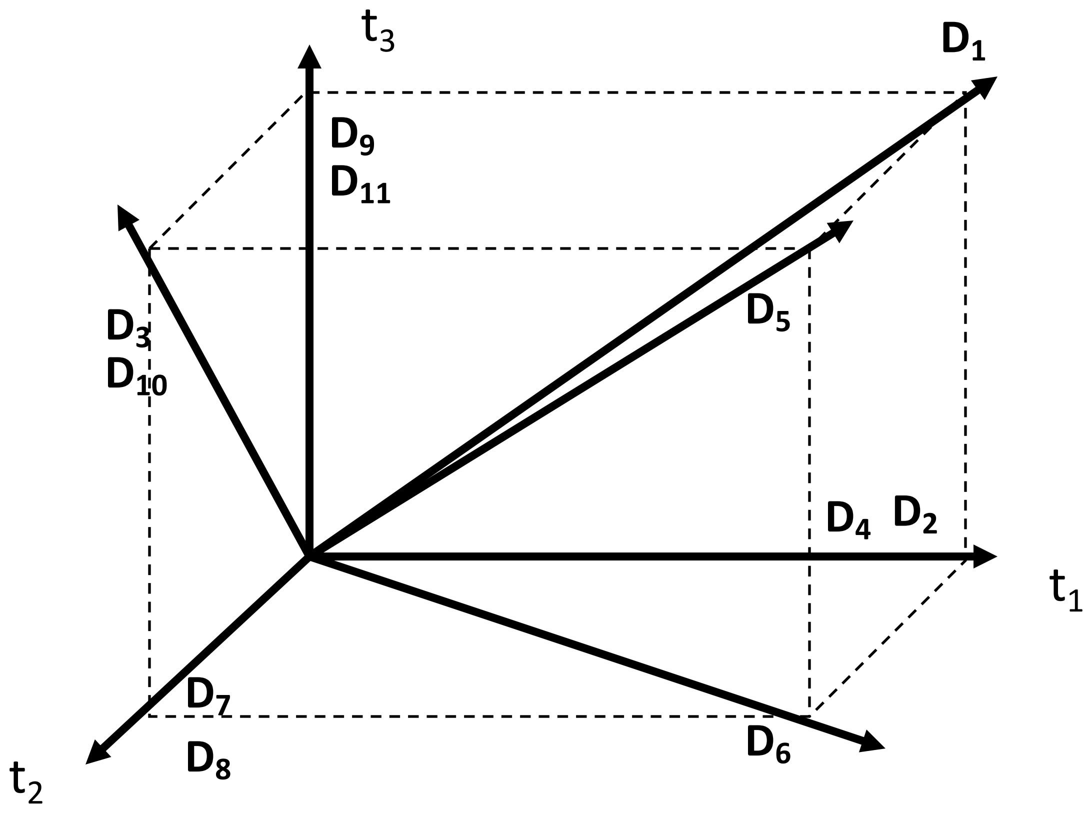
Vector space model (for |V|=3)



X, Y, Z are the 3 dimensions associated to keywords k_x, k_y, k_z

x, y, z are the 3 weights of keywords k_x, k_y, k_z in d_j

Documents in Vector Space



Vector Space Scoring Model

- Key idea 1: Do the same for queries: represent them as vectors in the space

- Key idea 2: Rank documents according to their proximity to the query in this space

- proximity = similarity of vectors

- proximity ≈ inverse of distance

- Recall: We do this because we want to get away from the you're-either-in-or-out Boolean model.

- Rank more relevant documents higher than less relevant documents

Vector Space Proximity

- First cut: distance between two points

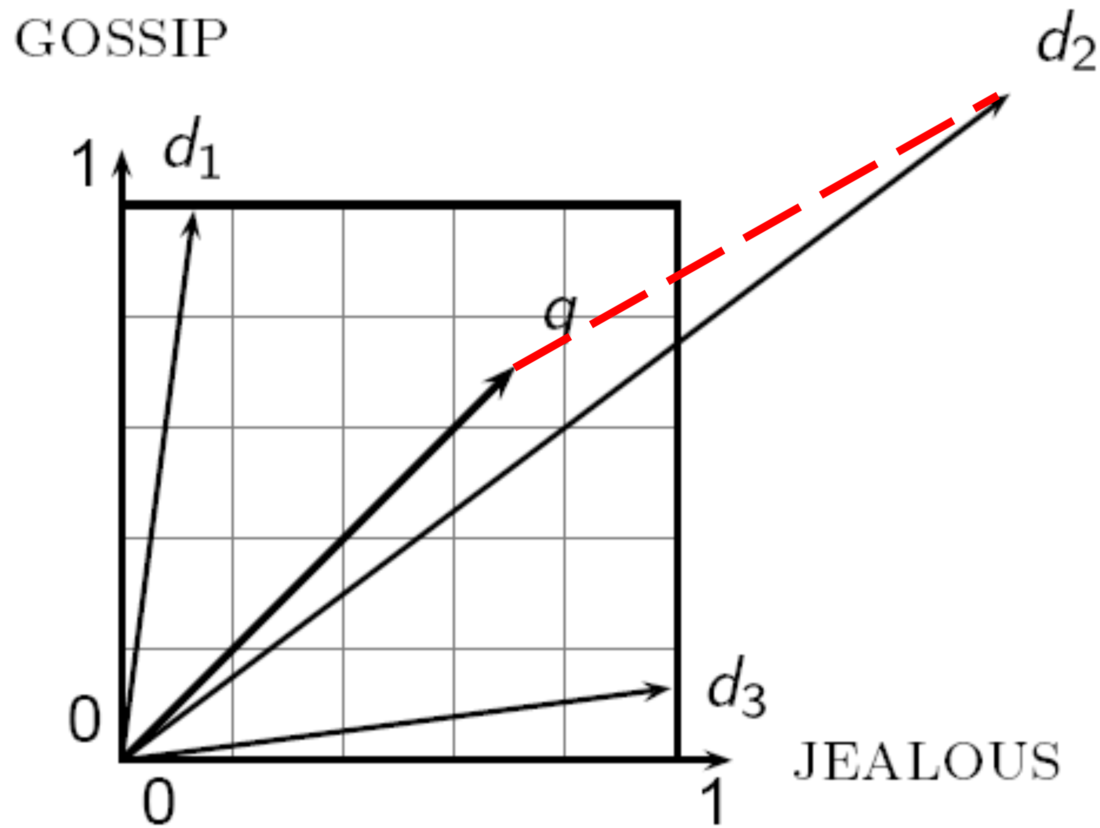
  (= distance between the end points of the two vectors)
- Euclidean distance?

$$d(d_j, q) = \sqrt{\sum_i (w_{ij} - w_{iq})^2}$$

- Euclidean distance is a bad idea . . .
- . . . because Euclidean distance is large for vectors of different lengths.

Why Euclidean distance is a bad idea?

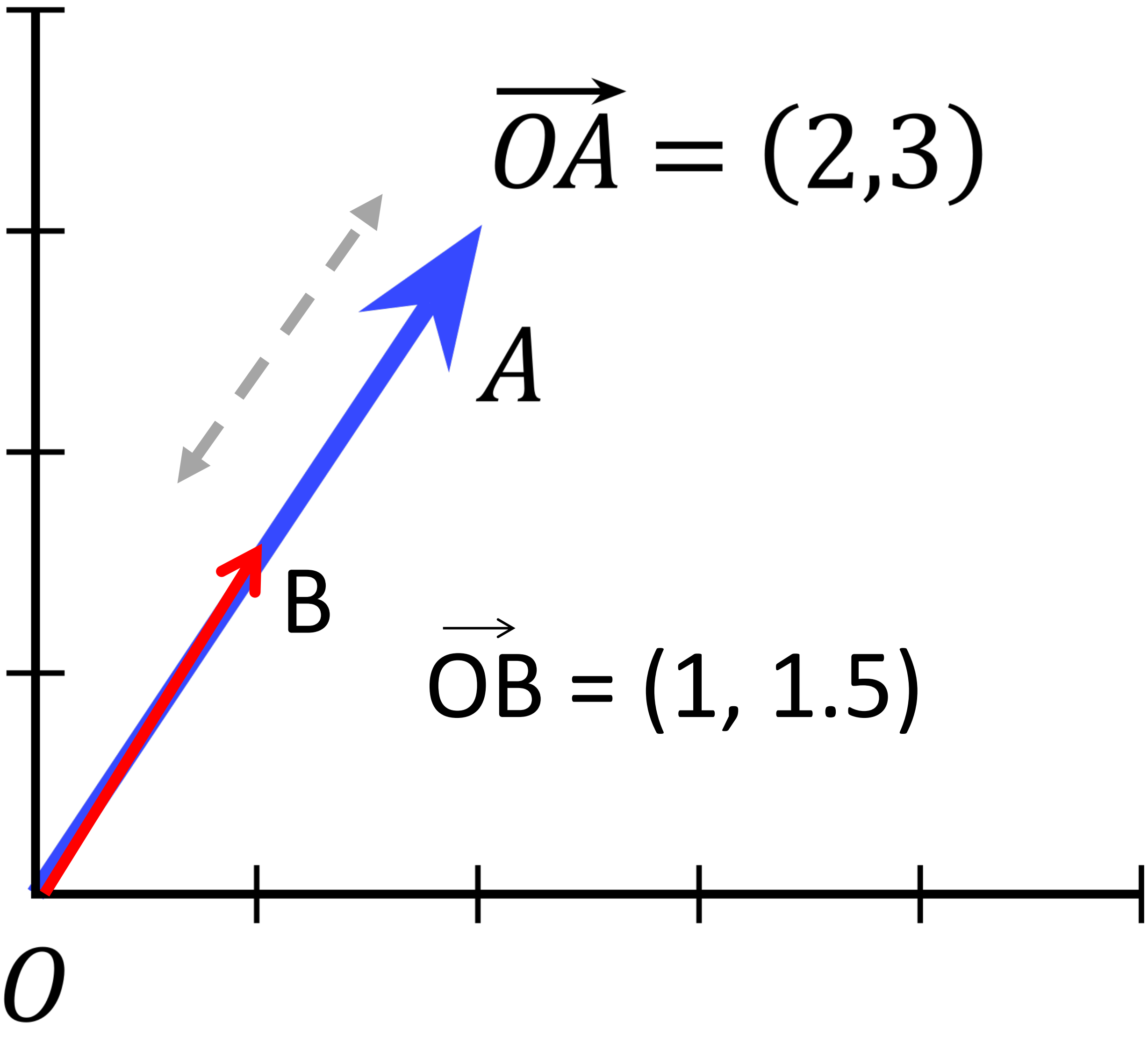
The Euclidean distance between $\vec{q}$ and $\vec{d_2}$ (red dashed line) is large even though the distribution of terms in the query $\vec{q}$ and the **distribution** of terms in the document $\vec{d_2}$ are very similar (about 50% gossip, 50% Jealous). Absolute frequencies cause the difference.



Why Euclidean distance is a bad idea?

- Experiment: take a document d and append it to itself. Call this document d’.
- “Semantically” d and d’ have the same content
- The Euclidean distance between the two documents can be quite large (word frequency doubles in d’)

Example



Measure the Angle between Documents

- In previous example, the angle between the two documents is 0.
- Key idea: Rank documents according to angle with query.
- In previous example, the angle is zero, corresponding to maximum similarity!
- In fact the two documents have the same words, with same relative weight.

From angles to cosines

- The following two notions are equivalent.
  - Rank documents in <u>decreasing</u> order of the angle between query and document
  - Rank documents in <u>increasing</u> order of **cosine**(query,document)

- Cosine is a monotonically decreasing function for the interval [0^o, 180^o]

Length normalization

- A vector can be (length-) normalized by dividing each of its components by its length – for this we use the L_2 norm:

$$\left\|\vec{x}\right\|_2 = \sqrt{\sum_i x_i^2}$$

  - Dividing a vector by its L_2 norm makes it a unit (length) vector
  - Effect on the two documents d and d′ (d appended to itself) from earlier slide:
    - they have **identical vectors** after length-normalization.

Vector Space Model: cosine-similarity

Dot product

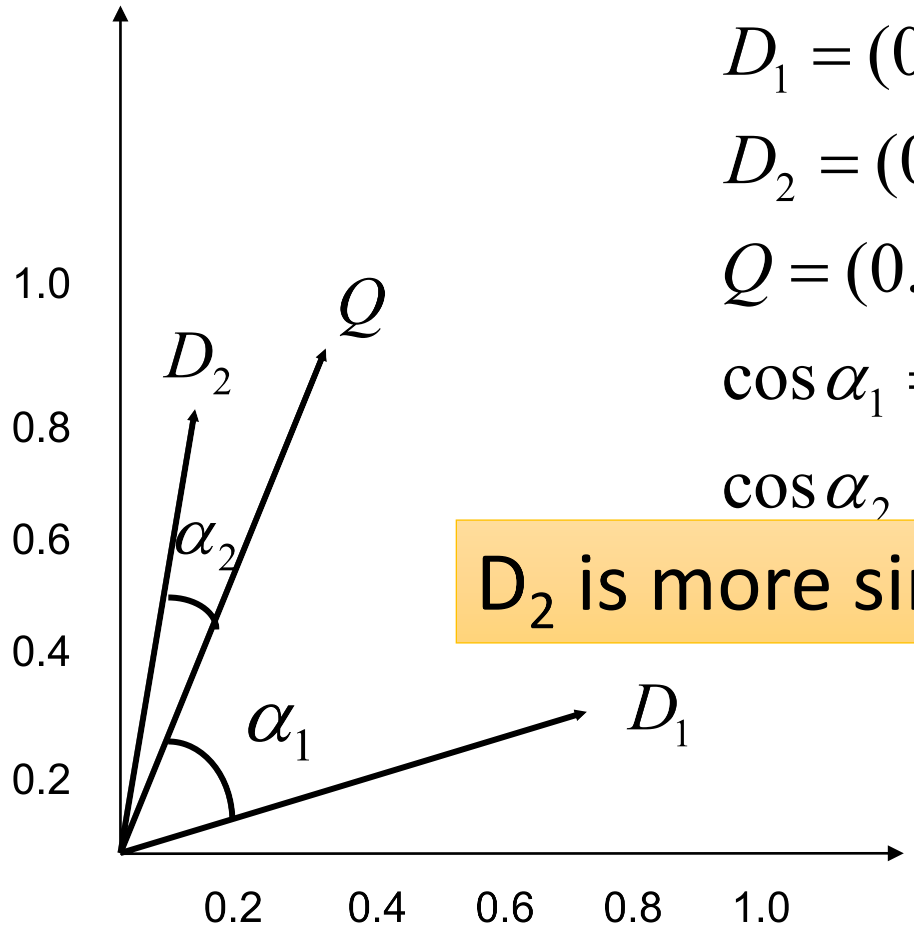
Unit vectors

$$\cos(\vec{q},\vec{d}) = \frac{\vec{q}\bullet\vec{d}}{|\vec{q}||\vec{d}|} = \boxed{\frac{\vec{q}}{|\vec{q}|}}\bullet\frac{\vec{d}}{|\vec{d}|} = \frac{\sum_{i=1}^{|V|} q_i d_i}{\sqrt{\sum_{i=1}^{|V|} q_i^2}\sqrt{\sum_{i=1}^{|V|} d_i^2}}$$

q_i is the tf-idf weight of term *i* in the query
d_i is the tf-idf weight of term *i* in the document
$\cos(\vec{q},\vec{d})$ is the cosine similarity of $\vec{q}$ and $\vec{d}$ … or,
equivalently, the cosine of the angle between $\vec{q}$ and $\vec{d}$.

Cosine-similarity is the cosine of the angle between normalized query end document vectors.

Example



$$D_1 = (0.8, 0.3)$$

$$D_2 = (0.2, 0.7)$$

$$Q = (0.4, 0.8)$$

$$\cos\alpha_1 = 0.74$$

$$\cos\alpha_2 = 0.98$$

D_2 is more similar to Q than D_1!!

A complete example

A small collection of N=3 documents, |V|=6 words

d1: "new york times"
d2: "new york post"
d3: "los angeles times"

Compute idf

angles	$log_2(3/1)=1.584$
los	$log_2(3/1)=1.584$
new	$log_2(3/2)=0.584$
post	$log_2(3/1)=1.584$
times	$log_2(3/2)=0.584$
york	$log_2(3/2)=0.584$

A complete example

Document-term matrix (we use **normalized tf**, however here each word appears just once in each document)

	angeles	los	new	post	times	york
d1	0	0	1	0	1	1
d2	0	0	1	1	0	1
d3	1	1	0	0	1	0

tf-idf: multiply tf by idf values

	angeles	los	new	post	times	york
d1	0	0	0.584	0	0.584	0.584
d2	0	0	0.584	1.584	0	0.584
d3	1.584	1.584	0	0	0.584	0

A complete example

Query: “new new times”

When computing the *tf-idf* values for the query terms we divide the frequency by the maximum frequency (2) to normalize, and multiply with the *idf* values

q	0	0	(2/2)*0.584=0.584	0	(1/2)*0.584=0.292	0

We calculate the length (the NORM) of each document vector and of the query:

Length of d1 = sqrt(0.584^2+0.584^2+0.584^2)=1.011
Length of d2 = sqrt(0.584^2+1.584^2+0.584^2)=1.786
Length of d3 = sqrt(1.584^2+1.584^2+0.584^2)=2.316
Length of q = sqrt(0.584^2+0.292^2)=0.652

A complete example

Similarity values are computed using cosin-sim formula:

$$\cos(\vec{q},\vec{d}) = \frac{\vec{q}\bullet\vec{d}}{|\vec{q}||\vec{d}|} = \frac{\vec{q}}{|\vec{q}|}\bullet\frac{\vec{d}}{|\vec{d}|} = \frac{\sum_{i=1}^{|V|} q_i d_i}{\sqrt{\sum_{i=1}^{|V|} q_i^2}\sqrt{\sum_{i=1}^{|V|} d_i^2}}$$

cosSim(d1,q) = (0*0+0*0+0.584*0.584+0*0+0.584*0.292+0.584*0) / (1.011*0.652) = 0.776
cosSim(d2,q) = (0*0+0*0+0.584*0.584+1.584*0+0*0.292+0.584*0) / (1.786*0.652) = 0.292
cosSim(d3,q) = (1.584*0+1.584*0+0*0.584+0*0+0.584*0.292+0*0) / (2.316*0.652) = 0.112

According to the computed similarity values, the final order in which the documents are presented as result to the query will be: d1, d2, d3.

Cosine Similarity

How similar are the novels:

SaS: *Sense and Sensibility*

PaP: *Pride and Prejudice*, and

WH: *Wuthering Heights*?

Cosine similarity amongst 3 documents

term	SaS	PaP	WH
affection	115	58	20
jealous	10	7	11
gossip	2	0	6
wuthering	0	0	38

Term frequencies (counts)

3 documents example contd

Log frequency weighting

term	SaS	PaP	WH
affection	3.06	2.76	2.30
jealous	2.00	1.85	2.04
gossip	1.30	0	1.78
wuthering	0	0	2.58

Tf-idf and normalize

term	SaS	PaP	WH
affection	0.789	0.832	0.524
jealous	0.515	0.555	0.465
gossip	0.335	0	0.405
wuthering	0	0	0.588

cos(SaS,PaP) ≈
$0.789 * 0.832 + 0.515 * 0.555 + 0.335 * 0.0 + 0.0 * 0.0$
≈ 0.94
cos(SaS,WH) ≈ 0.79
cos(PaP,WH) ≈ 0.69